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MULTI-OBJECTIVE OPTIMIZATION OF A THREE-ELEMENT TUNED MASS DAMPER FOR OFFSHORE PLATFORMS UNDER GROUND ACCELERATION

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Abstract

This paper presents a comparative study between the three-element tuned mass damper (T-TMD) and the conventional tuned mass damper (TMD) for vibration mitigation of offshore platforms subjected to ground excitations. The T-TMD is an enhanced configuration of the TMD, in which a viscous damper is connected in series with an additional spring. The equations of motion of offshore platforms equipped with the T-TMD are derived using the Lagrangian method. The constraint conditions of the T-TMD are analyzed to formulate the optimization problem, in which two objective functions are considered: maximizing vibration suppression efficiency and minimizing the device mass. The T-TMD parameters are optimized for a specific offshore platform based on these objectives. Numerical simulations demonstrate that the T-TMD provides superior performance and enhanced robustness compared to a conventional TMD of equal mass.

Keywords: *Multi-objective optimization, three-element TMD, passive control, offshore platforms, ground acceleration.*

1. Introduction

The application of tuned mass dampers (TMDs) for vibration control of offshore structures has been reported in several studies [1-4]. In recent years, an extended variant of TMD, known as the three-element tuned mass damper (T-TMD), has attracted growing attention. In the T-TMD configuration, a viscous damper is connected in series with an additional spring, introducing an extra degree of freedom associated with the displacement at the connection point between the damper and the secondary spring. This dual-spring arrangement provides enhanced vibration suppression performance and improves robustness against deviations in the structural natural frequency [5]. For the case of an undamped single-degree-of-freedom (SDOF) primary system, closed-form solutions for the optimal T-TMD parameters have been derived and experimentally validated [6]. For a damped SDOF system, approximate analytical approaches can be employed to determine the optimal parameters [7]. Previous studies [8, 9] have also demonstrated the effectiveness of the T-TMD in mitigating vibrations in both SDOF and multi-degree-of-freedom systems subjected to ground excitations.

In this study, the T-TMD is applied to suppress vibrations of an offshore platform under ground excitations. A simplified SDOF model of the offshore platform is considered. The equations of motion of the platform equipped with a T-TMD are derived using the Lagrangian method, and the T-TMD parameters are optimized using a genetic algorithm (GA). The optimization problem is formulated as a multi-objective framework with the goals of maximizing vibration suppression efficiency and minimizing the T-TMD mass. Finally, the performance of the T-TMD is compared with that of the conventional TMD through numerical simulations.

2. Analysis model and equations of motion

2.1. Analysis model

Consider the offshore platform modeled as an equivalent single-degree-of-freedom (SDOF) system, as illustrated in Fig. 1, with m_1, stiffness k_1, and damping c_1. The structure is equipped with a T-TMD device, as shown in Fig. 2.

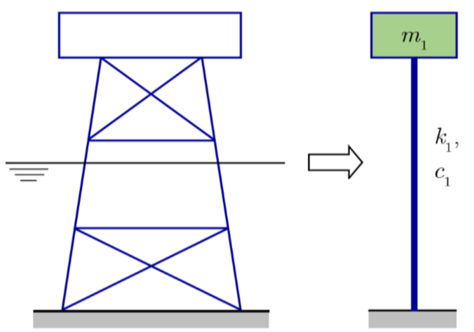

Figure 1. SDOF model of offshore platform

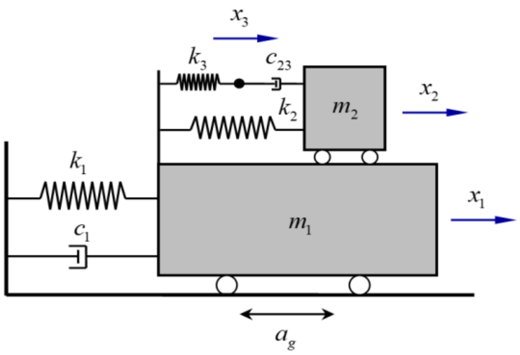

Figure 2. Schematic of offshore platform equipped with T-TMD

The structure is subjected to ground excitation with acceleration a_g. The T-TMD device consists of two springs with stiffness k_2 and k_3, and a viscous damper with damping coefficient c_{23}.

2.2. Equations of motion

Select the generalized coordinates:

$$\mathbf{q} = [x_1, x_2, x_3]^T \tag{1}$$

where x_1 denotes the displacement of the offshore platform, x_2 the displacement of the mass of the T-TMD, and x_3 the displacement of the connection point between the viscous damper and the second spring of the T-TMD. It should be noted that these coordinates are defined as absolute coordinates.

The kinetic energy of the system can be expressed as follows:

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 \tag{2}$$

The potential energy of the system is composed of the elastic potential energy of the structure and the T-TMD:

$$\Pi = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 \left(x_2 - x_1\right)^2 + \frac{1}{2} k_3 \left(x_3 - x_1\right)^2 \tag{3}$$

The dissipation function is defined as:

$$\Phi = \frac{1}{2} c_1 \dot{x}_1^2 + \frac{1}{2} c_{23} \left(\dot{x}_3 - \dot{x}_2\right)^2 \tag{4}$$

The system is subjected to ground excitation in the form of harmonic acceleration [8, 9]:

$$a_g = a_0 \sin \Omega t \tag{5}$$

In this case, the generalized forces of the non-conservative forces correspond to the inertia forces acting on m_1 and m_2:

$$\begin{aligned} Q_{x_1}^* &= -m_1 a_0 \sin \Omega t; \\ Q_{x_2}^* &= -m_2 a_0 \sin \Omega t; \\ Q_{x_3}^* &= 0 \end{aligned} \tag{6}$$

Applying the Lagrangian formulation, the system of equations of motion can be derived as follows:

$$\begin{aligned} & m_1 \ddot{x}_1 + c_1 \dot{x}_1 + \left(k_1 + k_2 + k_3\right) x_1 \\ & -k_2 x_2 - k_3 x_3 = -m_1 a_0 \sin \Omega t; \\ & m_2 \ddot{x}_2 + +c_{23} \dot{x}_2 - c_{23} \dot{x}_3 - k_2 x_1 \\ & +k_2 x_2 = -m_2 a_0 \sin \Omega t; \\ & -c_{23} \dot{x}_2 + c_{23} \dot{x}_3 - k_3 x_1 + k_3 x_3 = 0 \end{aligned} \tag{7}$$

3. Formulation of the optimization problem

In the steady-state condition, the solution of system (7) can be expressed in the form

$$\begin{aligned} x_1 &= A_1 \sin \Omega t + B_1 \cos \Omega t \\ x_2 &= A_2 \sin \Omega t + B_2 \cos \Omega t \\ x_3 &= A_3 \sin \Omega t + B_3 \cos \Omega t \end{aligned} \tag{8}$$

By substituting (8) into (7) and equating the coefficients of sinΩt and cosΩt, the following expressions are obtained:

$$\begin{cases} \left(-m_1 \Omega^2 + k_1 + k_2 + k_3\right) A_1 - c_1 \Omega B_1 \\ -k_2 A_2 - k_3 A_3 = -m_1 a_0; \\ c_1 \Omega A_1 + \left(-m_1 \Omega^2 + k_1 + k_2 + k_3\right) B_1 \\ -k_2 B_2 - k_3 B_3 = 0; \\ -k_2 A_1 + \left(-m_2 \Omega^2 + k_2\right) A_2 - c_{23} \Omega B_2 \\ +c_{23} \Omega B_3 = -m_2 a_0; \\ -k_2 B_1 + c_{23} \Omega A_2 + \left(-m_2 \Omega^2 + k_2\right) B_2 \\ -c_{23} \Omega A_3 = 0; \\ -k_3 A_1 + c_{23} \Omega B_2 + \left(-m_3 \Omega^2 + k_3\right) A_3 \\ -c_{23} \Omega B_3 = -m_3 a_0 \\ -k_3 B_1 - c_{23} \Omega A_2 + c_{23} \Omega A_3 \\ +\left(-m_3 \Omega^2 + k_3\right) B_3 = 0 \end{cases} \tag{9}$$

By solving system (9), the quantities A_i and B_i (i = 1, 2, 3) are obtained.

The vibration amplitude of the offshore platform can be expressed as:

$$x_{10} = \sqrt{A_1^2 + B_1^2} \tag{10}$$

In this case, the dynamic magnification factor of the structure is determined according to [8]:

$$DMF = \frac{\omega_1^2 x_{10}}{a_0} \tag{11}$$

where ω_1 is the natural frequency of the offshore platform, defined as:

$$\omega_1 = \sqrt{\frac{k_1}{m_1}} \tag{12}$$

Introducing the parameters:

$$\begin{aligned} &\omega_2 = \sqrt{\frac{k_2}{m_2}};\ \zeta_1 = \frac{c_1}{2m_1\omega_1};\ \zeta_{23} = \frac{c_{23}}{2m_2\omega_2}; \\ &\alpha = \frac{\omega_2}{\omega_1};\ \beta = \frac{k_3}{k_2};\ \mu = \frac{m_2}{m_1} \end{aligned} \tag{13}$$

Thus, for each mass ratio μ, it is necessary to determine the optimal of α, ζ_{23} and β such that the objective function [10] is satisfied:

$$\left(DMF_{\max}\right) \to \min \tag{14}$$

The design variables are considered within the ranges:

$$\begin{aligned} &\alpha \in \left[0.5 \div 1.5\right]; \\ &\zeta_{23} \in \left[0 \div 0.2\right]; \\ &\beta \in \left[0.1 \div 2.0\right] \end{aligned} \tag{15}$$

In the case of the structure equipped with a conventional TMD, as shown in Fig. 3, the TMD consists of a mass m_2, a spring with stiffness k_T, and a damper with damping coefficient c_T.

In this case, the following parameters are introduced:

$$\omega_2 = \sqrt{\frac{k_T}{m_2}};\ \zeta_T = \frac{c_T}{2m_2\omega_2};\ \alpha_T = \frac{\omega_2}{\omega_1} \tag{16}$$

The optimization problem then becomes determining the optimal values of α_T and ζ_T such that the objective function (14) is satisfied.

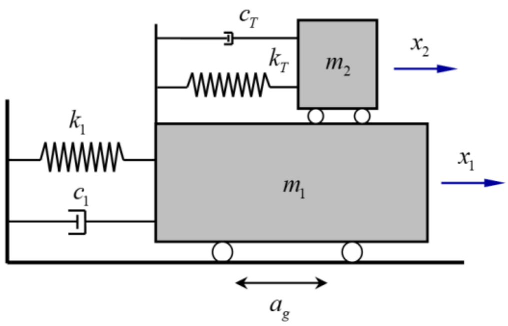

Figure 3. Schematic of offshore platform equipped with TMD

4. A case study

In the numerical model, a 249 m-high offshore platform installed in a water depth of 218 m is represented as a four-leg template structure. Through finite element analysis, the fixed platform is reduced from an MDOF representation to an equivalent SDOF system controlled by the fundamental mode [11]. The parameters of the fixed offshore platform are presented in Table 1.

Table 1. The parameters of the offshore platform [11]

Symbol	Value	Unit
m_1	7825307	kg
ω_1	2.048	rad/s
ζ_1	0.02	

In the absence of a damping device, the maximum dynamic magnification factor is DMF_{max} = 25. For a mass ratio varying from 0.5% to 2.5%, the optimal parameters of the T-TMD and TMD are determined using GA function in MATLAB. The GA was implemented with a population size of 50, a crossover rate of 0.8, a mutation rate of 1, and a stopping criterion set at 200 iterations. Table 2 presents the optimal parameter values of the T-TMD and TMD together with their corresponding DMF_{max} values. Figs. 4-8 illustrate the variation of the DMF with respect to the frequency ratio for both the optimal T-TMD and the optimal TMD.

It is observed that as μ increases from 0.5% to 2.5%, the vibration reduction efficiency improves; however, in all cases of mass ratio, the T-TMD outperforms the TMD. For the T-TMD, the optimal values of α^{opt} decrease, while the optimal values of ζ_{23}^{opt} and β^{opt} increase as μ varies from 0.5% to

Table 2. Optimal parameters of T-TMD and TMD for each given μ value

μ	T-TMD				TMD		
	$DMF_{\max}^{T-TMD}$	α^{opt}	ζ_{23}^{opt}	β^{opt}	$DMF_{\max}^{TMD}$	α_T^{opt}	ζ_T^{opt}
0.5%	11.605	0.950	0.083	0.198	11.743	0.990	0.046
1.0%	9.345	0.943	0.092	0.315	9.530	0.983	0.065
1.5%	8.133	0.929	0.110	0.383	8.352	0.976	0.081
2.0%	7.357	0.926	0.112	0.488	7.558	0.969	0.089
2.5%	6.778	0.914	0.126	0.555	6.989	0.962	0.099

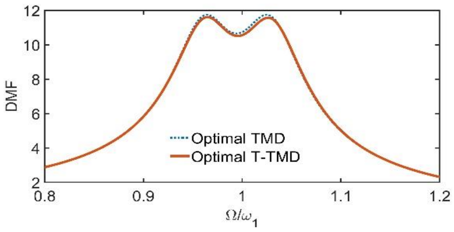

Figure 4. DMF curves corresponding to μ = 0.5%

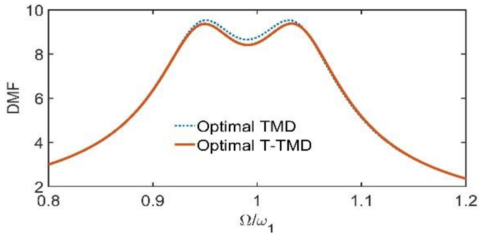

Figure 5. DMF curves corresponding to μ =1%

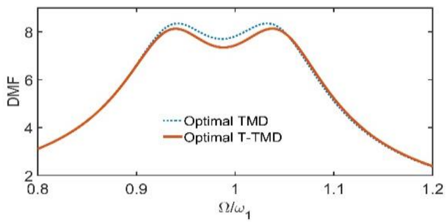

Figure 6. DMF curves corresponding to μ = 1.5%

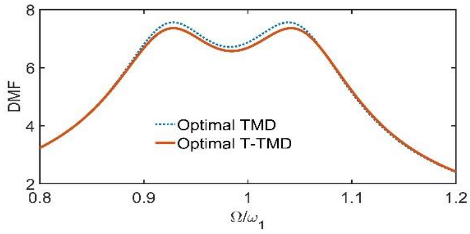

Figure 7. DMF curves corresponding to μ = 2%

2.5%. For the TMD, the optimal values of α_T^{opt} decrease, whereas the optimal values of ζ_T^{opt} increase with increasing mass ratio.

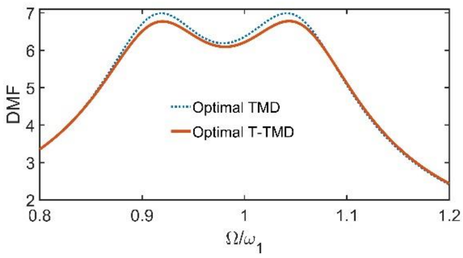

Figure 8. DMF curves corresponding to μ = 2.5%

5. Pareto front

The vibration mitigation performance of the device is evaluated through the parameter:

$$\gamma = \frac{DMF_{\max}^{UC} - DMF_{\max}^{T-TMD}}{DMF_{\max}^{UC}} \times 100\% \quad (17)$$

where DMF^{UC}_{max} denotes the maximum DMF_{max} of the offshore platform without a damping device, and $DMF^{T\text{-}TMD}_{max}$ represents the maximum DMF_{max} of the structure equipped with a T-TMD under optimal parameters.

Accordingly, the objective function (14) can be expressed in the form:

$$-\gamma \to \min \quad (18)$$

The multi-objective optimization problem involves two objectives: maximizing the vibration mitigation efficiency while minimizing the utilized mass. The condition of minimizing the utilized mass is expressed as:

$$\mu \to \min \quad (19)$$

However, by applying the Epsilon-constraint method Epsilon [12], the objective function (19) is transformed into a constraint:

$$\mu \le \varepsilon \tag{20}$$

With ε considered in the range [0.5% ÷ 2.5%].

Fig. 9 illustrates the variation curve of γ with respect to the values of μ. This presents the Pareto front of the objective functions (18) and (19).

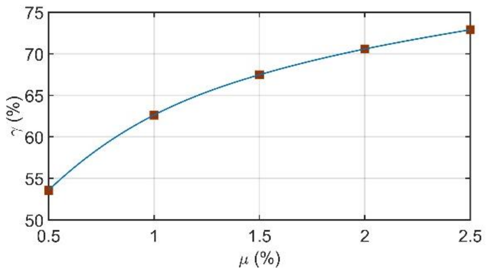

Figure 9. Variation of γ with μ for the case of T-TMD installation

6. Robustness

The actual structural parameters may differ from the design values due to various factors, such as uncertainties in material properties [13]. In addition, offshore platforms are also affected by marine fouling. Therefore, it is necessary to investigate the effectiveness and robustness of T-TMT and TMD under varying structural parameters, particularly the stiffness parameter.

Introducing the parameter:

$$\Delta k_1 = \frac{k_a - k_1}{k_1} \tag{21}$$

Where k_a denotes the actual stiffness of the structure, and Δk_1 represents the deviation of the actual stiffness from the design stiffness.

Table 3. DMF_{max} values corresponding to Δk_1

Δk_1	DMF_{max}	
	T-TMD	TMD
-25%	19.607	20.515
-20%	16.606	17.407
-15%	13.766	14.411
-10%	11.228	11.700
-5%	9.083	9.403
0	7.357	7.558
+5%	8.212	8.448
+10%	9.057	9.310
+15%	9.848	10.107
+20%	10.560	10.806
+25%	11.181	11.404

The values of Δk_1 are considered within the range of [-25%÷25%] with an increment 5%. The corresponding DMF_{max} values for the structures equipped with T-TMD and TMD are obtained for each Δk_1, as shown in Table 3. It is observed that in all cases, T-TMD outperforms TMD, particular under unfavorable conditions when the actual stiffness tends to decrease relative to the design stiffness.

Figs. 10 and 11 illustrate the variation of *DMF* with respect to the values Δk_1, corresponding to the cases of T-TMD and TMD installation, respectively.

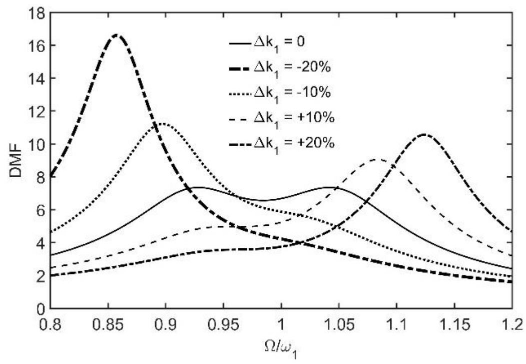

Figure 10. DMF curves for different Δk_1 values with T-TMD

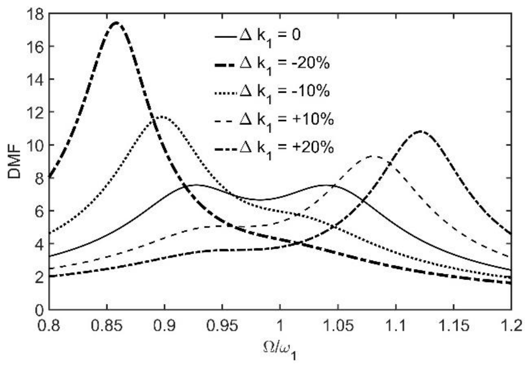

Figure 11. DMF curves for different Δk_1 values with TMD

The three-element TMD (T-TMD) exhibits better robustness than a conventional TMD against changes in the structural stiffness because its additional elements allow energy dissipation over a wider frequency range, reducing sensitivity to variations in the system's natural frequency.

7. Conclusion

This paper presents the application of a T-TMD device to mitigate vibrations of an offshore platform, modeled as a SDOF system, subjected to harmonic ground acceleration. The main novelties of this study include the use of the well-known Genetic Algorithm (GA) in MATLAB to determine the optimal parameters of the T-TMD, the application of the ε-constraint method to address the multi-objective optimization problem, and the establishment of a Pareto front between vibration reduction and mass utilization, allowing evaluation of the trade-off between these objectives. The robustness of T-TMD and TMD was compared under variations of structural stiffness for a mass ratio of $\mu = 2\%$. Results indicate that the T-TMD exhibits higher robustness than the TMD, particularly when the stiffness decreases.

Future research will extend this work to offshore platforms modeled as multi-degree-of-freedom systems subjected to stochastic ground excitations, such as earthquake loads.

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