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RELIABILITY-BASED DESIGN OPTIMIZATION OF STRUCTURES: AN INVESTIGATION ON PLANAR STRUCTURES

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Abstract

Structural optimization is a crucial approach for achieving rational design solutions, in which cost is commonly set as the objective to be minimized. Two primary approaches are typically employed: deterministic-based optimization and reliability-based optimization. To address the inherent uncertainties in structural performance, this study proposes a procedure for optimizing structures to satisfy both deterministic and probabilistic constraints. In this framework, the problem is formulated to incorporate probabilistic safety requirements, and Monte Carlo simulation is used to estimate the failure probability at each iteration of the optimization process. Three illustrative examples are presented to demonstrate the effectiveness of the proposed procedure for planar structures. The results from the reliability-based approach are examined for different levels of probabilistic constraints, and the influence of including uncertain variables is investigated in the first two examples. The findings show that the reliability-based approach generally results in larger element sections and, consequently, higher investment costs when lower target failure probabilities are specified, and greater weight is required when additional uncertain variables are considered. Finally, the proposed procedure is shown to perform well for problems defined implicitly and involving both deterministic and probabilistic constraints, as evidenced in Example 3.

Keywords: *Reliability-based design optimization, Monte Carlo simulation, structural optimization, particle swarm optimization.*

1. Introduction

In structural engineering practice, optimization-based design has been widely adopted to improve efficiency and reduce costs [1-3]. Structural optimization provides feasible design solutions by minimizing the investment cost, which is often represented by the total weight or volume of the structure [4, 5]. The fundamental idea of optimization arises from the fact that an infinite number of design configurations can satisfy the basic design requirements. For instance, when designing a beam to safely resist an applied load, both the width and the height of its cross-section must exceed certain minimum values to ensure sufficient capacity. However, countless combinations of width and height can satisfy this condition. Optimization methods allow engineers to identify not only a safe design but also the minimum cost (e.g., material usage) [6].

Based on this concept, optimization can be conducted under deterministic constraints (Deterministic-Based Design Optimization, DBDO) or under probabilistic constraints (Reliability-Based Design Optimization, RBDO). The DBDO approach ensures safety requirements, such as strength or serviceability, in a deterministic manner [6]. In contrast, RBDO extends this framework by incorporating uncertainties and ensuring that the optimized solution satisfies a prescribed reliability level. It has been demonstrated that each design solution is inherently associated with a certain probability of failure; hence, DBDO may fail to meet a target reliability index because probabilistic constraints are not explicitly considered [7].

The general formulation of RBDO is given in Eq. (1). The optimization seeks to minimize the cost function *C*, typically represented by structural weight or volume (Eq. (1a)). Deterministic requirements are imposed through Eq. (1b), while Eq. (1c) introduces the probabilistic constraints. In this formulation, the design variables *x*, which belong to the design space *D*, are optimized such that both deterministic and probabilistic constraints are satisfied while minimizing the cost function [8].

$$\min C = \arg\min_{x \in D} C\left(x\right) \tag{1a}$$

$$s.t. \qquad f(x) \le f_0 \tag{1b}$$

$$s.t. \qquad P_f \le P_{f,T} \tag{1c}$$

Since RBDO imposes additional probabilistic constraints, the resulting optimal cost is generally higher than that obtained from DBDO. Nevertheless, RBDO ensures that the final design satisfies prescribed reliability levels, thereby providing a more uniform and consistent design outcome. It should also be noted that the computational demand of RBDO is significantly higher than that of DBDO due to the need for repeated reliability analyses.

In this study, the concept and procedure of RBDO are presented and applied to structural optimization problems. Section 2 introduces the methodology for implementing RBDO. Section 3 demonstrates the application through three illustrative examples to examine the optimization process and its outcomes. Finally, Section 4 summarizes the findings and provides concluding remarks.

2. Reliability-based design optimization for planar structures

Different from deterministic-based design optimization, the constraints in RBDO are represented by probabilistic measures, such as target reliability indices or target probabilities of failure. Therefore, reliability analyses must be embedded into the optimization procedure, which substantially increases the computational effort. Nonetheless, the obtained design tends to guarantee the specified reliability requirements. Any optimization algorithm can be used in RBDO. Gradient-based methods are preferred for explicit performance functions, whereas metaheuristic algorithms are suitable for implicit ones. The general RBDO procedure is outlined in this section.

2.1. Reliability assessment

To evaluate probabilistic quantities such as the reliability index or the probability of failure associated with a trial design parameter vector x in Eq. (1), various reliability analysis methods can be applied. These methods are well documented in the literature (e.g., Haldar & Mahadevan, 2000; Hurtado, 2004). In this study, the Monte Carlo Simulation (MCS) method is adopted due to its straightforward implementation and its ability to handle both explicit and implicit performance functions. Although the fundamentals of MCS have been extensively presented in the literature, e.g., [10-12], its main steps are summarized for clarity.

Given a limit state function $g(u)$, which is formulated from a performance function $p(u)$ as shown in Eq. (2a), MCS estimates the probability of failure, i.e., the probability (P) that $p(u)$ exceeds a prescribed threshold p_0, as expressed in Eq. (2b).

$$g(u) = p(u) - p_0 \tag{2a}$$

$$P\left[g(x) < 0\right] = P\left[p(x) < p_0\right] \tag{2b}$$

In these equations, u denotes a vector of both deterministic and random variables. In structural reliability analysis, $g(u)$ represents a performance criterion related to strength or serviceability that must be satisfied during the design process. For instance, $g(u)$ can be expressed as $g(u) = R - Q$, where R is the resistance and Q is the applied load. Alternatively, $g(u)$ may be formulated in terms of the safety factor, e.g., $g(u) = FS - 1$ [10].

The procedure of MCS can be summarized in the following steps [10]:

Step 1. Sampling random variables. A set of realizations of the input random variables is generated based on their statistical information.

Step 2. Evaluating performance and limit state functions. For each sample, the performance function $p(u)$ is evaluated. In structural problems, $p(u)$ is often determined implicitly and may require numerical procedures such as the finite element method. The corresponding limit state function $g(u)$ is then computed.

Step 3. Repetition for all samples. In this step, Step 2 is repeated for the entire set of samples generated in Step 1.

Step 4. Identifying failure events. Failure occurs when $g(u) < 0$. The total number of failure samples is recorded.

Step 5. Estimating the probability of failure. The probability of failure, P_f, is estimated as the ratio of failure events to the total number of generated samples.

Step 6. Computing the reliability index. The corresponding reliability index β is then obtained from Eq. (3):

$$\beta = -\Phi^{-1}\left(P_f\right) \tag{3}$$

Where Φ is the standard normal cumulative distribution function.

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The accuracy of MCS strongly depends on the number of samples used in Step 1. It is generally recommended that at least 10^6 samples be employed when assessing reliability indices up to 4.0 [10]. Since structural reliability indices often fall within the range of 2.5-4.0, a large sample size is usually required for MCS-based analyses. However, the computational demand of MCS is high, especially when implicit limit state functions require finite element evaluations. To enhance efficiency, variance reduction techniques and advanced simulation strategies can be implemented [13].

2.2. Optimization algorithm

Optimization methods are generally classified into two categories: gradient-based and metaheuristic algorithms. Gradient-based methods use derivatives to guide the search toward the optimum of the objective function. Common examples include Gradient Descent, Newton's method, the Conjugate Gradient method, and Sequential Quadratic Programming (SQP). Because derivative information is required, these algorithms are best suited for optimization problems with explicitly defined and differentiable objective and constraint functions. When derivatives are not analytically available, numerical approximations such as finite-difference schemes may be used.

Metaheuristic optimization, in contrast, does not require derivative information. These algorithms are based on stochastic or nature-inspired search strategies and can be applied to a broader range of problems, including nondifferentiable, discontinuous, or highly nonlinear functions. Representative methods include Genetic Algorithms, Simulated Annealing, Ant Colony Optimization, Artificial Bee Colony, and Particle Swarm Optimization (PSO) [14]. A key advantage of metaheuristic algorithms is their capability to explore the design space more globally and handle functions with multiple local optima.

In this work, the *fmincon* function, available in MATLAB (MATWORKS), is employed for explicit objective and performance functions. This function supports several gradient-based algorithms such as SQP, interior-point, active-set, and trust-region-reflective methods. Notably, SQP and interior-point are effective for problems with nonlinear constraints, whereas active-set is more suitable for problems with linear constraints.

For problems with implicitly defined performance functions, the built-in MATLAB function *particleswarm* is utilized (MATWORKS). The *particleswarm* function implements the PSO algorithm, a metaheuristic method that searches for the global minimum of a nonlinear objective function within a bounded design space. It is particularly advantageous for problems that are nonsmooth, non-differentiable, or stochastic in nature. Unlike gradient-based methods such as *fmincon*, *particleswarm* does not require derivative information. The algorithm simulates a swarm of particles, each representing a candidate solution that updates its position based on individual and shared experiences. MATLAB also provides flexibility for parameter tuning (e.g., swarm size, number of iterations, inertia weight, and convergence criteria) and supports parallel computing, which significantly improves computational efficiency.

To incorporate reliability constraints into the optimization algorithms, the probabilistic requirement in Eq. (1c) can be expressed as an inequality within the constraint function of *fmincon*. Alternatively, when *particleswarm* is used, the original objective function C in Eq. (1a) can be reformulated into a penalized objective function C_{pen} to account for the constraints in Eq. (1), as expressed in Eq. (4).

$$C_{pen} = C + M * \max\left(0, f\left(x\right) - f_0\right) + M * \max\left(0, \frac{\left|P_f - P_{f,T}\right|}{P_{f,T}} - 0.01\right) \quad (4)$$

In Eq. (4), M denotes the penalty term, which is assigned a sufficiently large value (e.g., 10^4) when either Eqs. (1b) or (1c) is violated, and zero otherwise. In other words, no penalty is imposed if all deterministic and probabilistic constraints are satisfied.

2.3. Flowchart of RBDO for planar structures

Unlike traditional DBDO, the RBDO process involves two nested loops, which results in higher computational demand. In the outer loop, the design variables are iteratively updated within the design space, while in the inner loop, the probabilistic constraints (e.g., Eq. 1(c)) are evaluated. The general procedure can be summarized as follows:

Step 1. Problem definition. Define the design variables and their bounds, the objective function, and the deterministic, probabilistic constraints. For MCS-based reliability analysis, the random variables and their statistical properties must also be specified.

Step 2. Generation of trial design variables. If *fmincon* is applied, trial designs are generated starting from the initial guess. When *particleswarm* is used, trial designs are randomly sampled from the design space according to the PSO algorithm.

Step 3. Deterministic evaluation. Compute the objective function and check the deterministic constraints for each trial design.

Step 4. Probabilistic evaluation. For probabilistic constraints, perform reliability analysis to estimate the probability of failure, P_f. Compare the estimated P_f with the target value specified in Eq. (1c). If violated, apply a penalty function or constraint handling scheme. The response surface method may be incorporated to reduce the computational cost of the reliability analysis [16].

Step 5. Convergence check and update. Assess whether the convergence criteria are satisfied (e.g., small change in objective function, design variables, or reliability index). If not, update the design variables according to the optimization algorithm and repeat the process.

Step 6. Output of optimal design. Once convergence is achieved, report the optimal design variables, the minimized objective function, and the associated reliability performance.

3. Illustrative examples

This section presents three illustrative examples to demonstrate the proposed procedure. The first example examines the plastic capacity of a portal frame, where only the cross-sectional dimensions of the frame columns are treated as uncertain variables. The second example extends the first by investigating the effect of treating the applied load as an additional source of uncertainty. Finally, the third example addresses the serviceability performance of a truss. Whereas the first two examples emphasize the influence of uncertainty levels on the resulting design solutions, the third example highlights the application of the proposed framework to an implicitly defined problem that incorporates both deterministic and probabilistic constraints.

3.1. Example 1 - a portal frame without uncertainty in the applied load

In this example, a portal frame subjected to a horizontal load is considered, as illustrated in Fig. 1. This frame configuration was previously introduced in an earlier study [17]. However, only the reliability assessment of the frame was presented in the work, which reported a failure probability of 0.0092. This value is adopted as a benchmark for the initial condition of the frame to evaluate the accuracy of the MCS-based implementation in the present study. In this work, the RBDO of the frame is investigated.

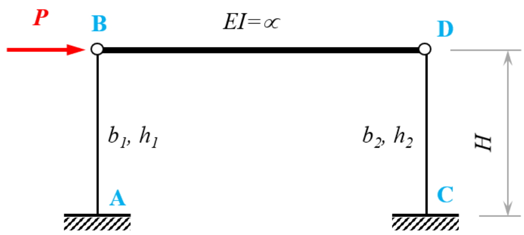

Figure 1. A portal frame subjected to a horizontal load

In the design process, the rectangular sections of the two columns, represented by the width and the height of the section (b, h), are designed. Both the frame height (H) and the applied load (P) are treated deterministically with a certain value of 1m and 4 kN. On the other hand, the dimensions of column sections are assumed to be the uncertain variables. The statistical properties of b and h are taken from the previous study, as summarized in Table 1.

Table 1. Statistical properties of the variables

Variable	μ (cm)	COV	Type of Dist.
b_1	10	0.2	Normal
h_1	12	0.2	Normal
b_2	10	0.2	Normal
h_2	12	0.2	Normal

The limit state is defined by the formation of a plastic hinge at the bottom of the left column. Consequently, the reliability analysis reduces to assessing the failure probability associated with the occurrence of a plastic hinge at this location. Using structural mechanics principles, the moment capacity at the bottom of the left column, R, and the corresponding resultant moment S at this section are expressed in Eqs. (5a) and (5b), respectively [17]. Based on these definitions, the limit state function is formulated in Eq. (6a), and the corresponding failure probability can be evaluated using Eq. (6b) in reliability analyses.

$$R = \frac{1}{3}\left[\sigma\right]b_1h_1^2 \tag{5a}$$

$$S = \frac{I_1}{I_1 + I_2}PH = \frac{b_1h_1^3}{b_1h_1^3 + b_2h_2^3}PH \tag{5b}$$

$$g = R - S \tag{6a}$$

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$$P_f = P\left(g = R - S < 0\right) \qquad (6b)$$

In the RBDO, this problem can be defined as minimizing the weight of the two columns such that the P_f is not larger than a target $P_{f,T}$. The width and height of the column sections are assumed to range from 5cm to 30cm. Thus, the RBDO can be defined by Eq. (7), noting that the volume and the weight of the column can be interchanged. For the MCS-based reliability analysis, 1 million simulations are conducted, and *fmincon* is applied to this example.

$$\min\left(V_{col.}\right) = \arg\min_{b_1,b_2,h_1,h_2}\left(b_1h_1 + b_2h_2\right) \qquad (7a)$$

$$s.t. \qquad P_f \le P_{f,T} \qquad (7b)$$

$$\text{and} \qquad \begin{matrix} 5cm \le b_1, b_2 \le 30cm \\ 5cm \le h_1, h_2 \le 30cm \end{matrix} \qquad (7c)$$

In this example, various targets of P_f, including 1×10^{-2}, 5×10^{-2}, and 10×10^{-2}, are used to investigate the outcome of the optimal design solutions.

Using the procedure outlined in Section 2, the results of RBDO for the three thresholds of $P_{f,T}$ are summarized in Figs. 2, 3, and 4. It is seen in the figures that the objective functions exhibit a decreasing trend by iteration. The optimal areas are indicated by the green markers and the legends in the upper panels of the three figures. The probabilistic constraints are also reported during the optimization process in the lower panels. It is observed in the figures that the probabilistic assessments tend to vary around the probabilistic constraint specified, and the probabilistic assessments are all achieved at the final iteration.

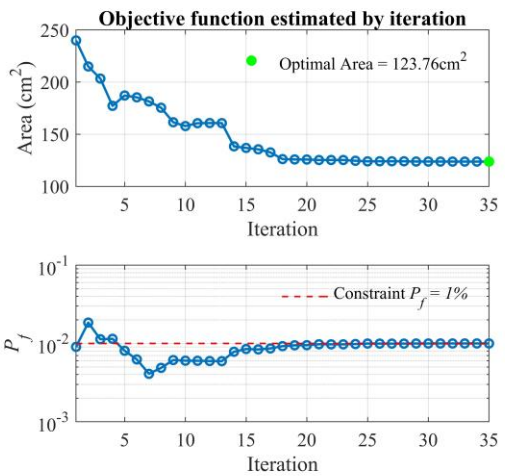

Figure 2. RBDO for Example 1 with $P_{f,T}$ = 1%

Moreover, the figures indicate that larger cross-sections do not necessarily guarantee safer designs in terms of failure probability. For example, at the second iteration in Fig. 2, the two columns have a combined area of approximately 215cm^2, yet the corresponding failure probability is about 1.85%, which is higher than that obtained at the final iteration.

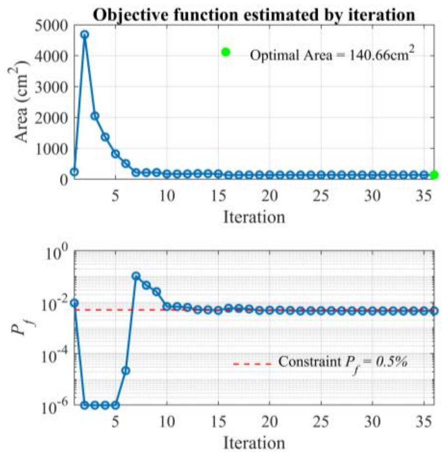

Figure 3. RBDO for Example 1 with $P_{f,T}$ = 0.5%

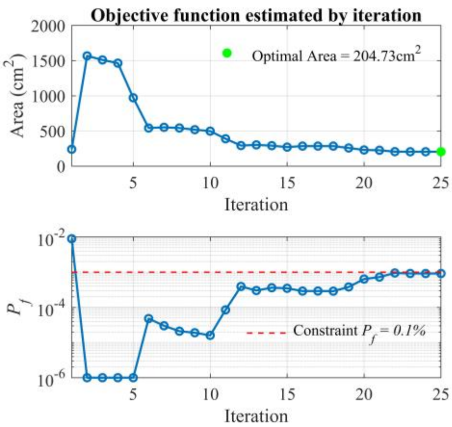

Figure 4. RBDO for Example 1 with $P_{f,T}$ = 0.1%

Finally, the results in the three figures emphasize that larger section areas are required for lower thresholds of Pf,T. For instance, when the failure probability decreases ten times (from 1% to 0.1%), the section area might increase 65% (from 123.76cm^2 to 204.73cm^2). That is, the results of RBDO significantly depend on the threshold of Pf specified, which might not be accounted for in the DBDO.

3.2. Example 2 - a portal frame with uncertainty in the applied load

In this example, the portal frame examined in Example 1 is reconsidered, wherein the load is treated as an uncertain variable. When the load is simulated as an uncertain variable, a COV of 0.25 is assumed. This investigation provides insight into the effect of the random variables. Notably, the width and the height of column sections are also the optimized variables.

Fig. 5 shows the results for the threshold $P_{f,T}$ of 1%. It is seen in the figure that the convergence is achieved, and the threshold $P_{f,T}$ is also satisfied at the final iteration. The optimal area is 214.39cm^2, corresponding to design variables of 5.1 cm, 15.2cm, 5cm, and 27.4cm. This result is slightly higher than that reported in Fig. 4 (204.73cm^2). The result in Example 2 is higher than that of Example 1 by about 5%, indicating the effect of random consideration of load in RBDO.

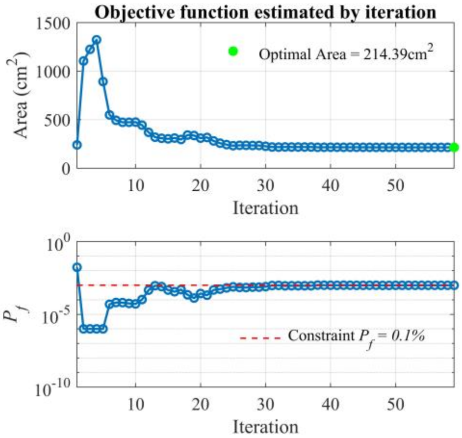

Figure 5. RBDO for Example 2 with $P_{f,T}$ = 0.1%

3.3. Example 3 - a planar truss

In this example, a Pratt truss, examined in previous works [18, 19], is studied. The truss consists of 13 bars connected via 8 nodes, as shown in Fig. 6. The truss is subjected to a horizontal load P at Node 6. The uncertain variables in this example include the cross-sectional area (A) and the applied load (P). For convenience, the truss members are classified into three groups: lower chords, upper chords, and diagonal bars. The cross-sectional areas of the members in each group are assumed to be fully independent. The design optimization aims to minimize the total weight of the truss; accordingly, the objective function is formulated in Eq. (8). The strength constraint, ensuring the structural safety of the truss, is expressed in Eq. (9a).

$$\min\left(V\right) = \arg\min_{A_i}\left(\sum_{1}^{13} A_i L_i\right) \tag{8}$$

$$\sigma_i \leq \left[\sigma\right], \qquad \forall i \in \left(1 \to 13\right) \tag{9a}$$

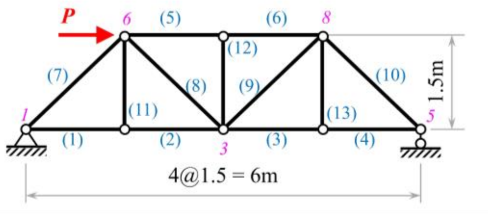

Figure 6. Example 3 - A truss structure

In the RBDO formulation, the truss is designed such that the probability of the drift exceeding 5 mm is no greater than 5×10^{-2}. Accordingly, besides the strength constraint in Eq. (9a), the serviceability constraints for RBDO are written in Eqs. (9b).

$$P\left(x_6 > 5mm\right) \leq 10^{-3} \tag{9b}$$

At the initial condition, the lower, upper, and diagonal cross-sectional areas are 2cm^2, 4cm^2, and 1.5 cm^2, respectively. The lower and upper bounds for the search are set to 0.2 and 2 times the initial cross-sectional area of each member group, respectively. Notably, the *particleswarm* function is employed in this example because the structural response is evaluated through an implicit process, as addressed in Section 2. The *FEM-Truss* program, previously developed and applied in our works [10, 11], is used here to compute the truss responses. The response surface method is also implemented to reduce the computational burden. For the MCS-based reliability analysis in this example, 200,000 samples are employed, and *particleswarm* is conducted.

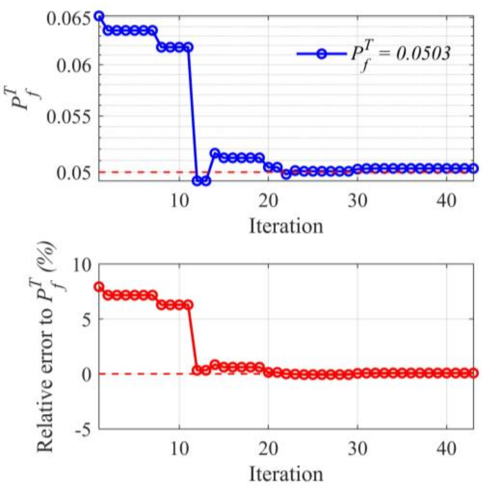

Figure 7. Convergence of P_f to the target of $P_{f,T}$ = 5% for Example 3

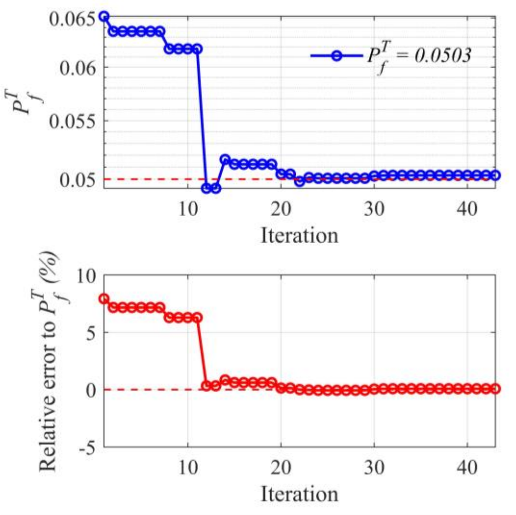

Figure 8. Convergences in Example 3

Following the flowchart presented in Section 2, the results of Example 3 are shown in Figs. 7 and 8. Particularly, Fig. 7 illustrates the estimated failure probabilities over the iterations. The optimization converges after 43 iterations, as indicated by a small relative error with respect to the target probability of only 0.09% at the final iteration. Fig. 8 presents the penalty objectives (Eq. (4)) together with the total cross-sectional area of the truss (i.e., the total volume in Eq. (8)). In the initial iterations, the penalty objective function is significantly larger than the true objective function, implying constraint violations. After about 16 iterations, the penalty objective becomes identical to the true objective, indicating that all constraints are satisfied. Finally, the minimum volume of the truss is determined to be 95.64cm^2, corresponding to the optimal cross-sectional areas of 2.406cm^2, 2.535cm^2, and 1.109cm^2 for the lower chords, upper chords, and diagonal bars, respectively.

4. Conclusions

This study presents a procedure for the optimal design of structures under probabilistic assessment conditions. The probabilistic requirements are formulated as constraints, and two optimization algorithms are employed to address both explicit and implicit problems. Based on three illustrative examples, several important conclusions can be drawn as follows.

Investigations in Example 1 demonstrate that the design solutions in RBDO strongly depend on the specified target failure probability. For instance, when the target failure probability decreases by an order of magnitude (from 1% to 0.1%), the cross-sectional area increases by approximately 65% (from 123.76 cm² to 204.73 cm²). This observation highlights that the results of RBDO are highly sensitive to the prescribed P_f threshold, whereas DBDO might remain unaffected. Notably, lower target failure probabilities lead to greater material usage under the same uncertainty conditions in practice.

Example 2 shows that, for the same target P_f of 0.1%, the material usage increases by about 5% compared with Example 1 when the load is treated as an uncertain variable. This emphasizes that in RBDO, the design results are significantly influenced by the uncertainties considered, and the appropriate inclusion of uncertain variables governs the reliability of the optimal solution—an aspect not accounted for in traditional DBDO.

Finally, Example 3 illustrates that the proposed procedure performs well for problems defined implicitly and simultaneously subjected to deterministic and probabilistic constraints, further emphasizing the applicability of the proposed framework.

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